Quasi-integrable non-linear Schrodinger models
and infinite towers of conserved charges for dark and
bright solitons

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Abstract

Deformations of the focusing and defocusing non-linear Schrodinger models (NLS) are considered in the context of the quasi-integrability concept. We strengthen the results of JHEP09(2012)103 for bright soliton collisions. The both (deformed) focusing and defocusing NLS’s (the defocusing case has been presented in JHEP03(2016)005, and the focusing case in JHEP05(2017)106) exhibit an infinite tower of exactly conserved charges. We show, by means of analytical and numerical methods, that for certain two-soliton (bright or dark) solutions, in which the modulus and phase of the complex modified NLS field exhibit definite parities under a space-reflection symmetry, the first four and the sequence of even order charges are exactly conserved during the scattering process of the solitons. We perform extensive numerical simulations and consider the bright solitons with focusing deformed potential \( V = \frac{2\eta}{2+i\epsilon} |\psi|^2 \), \( \eta < 0, \epsilon \in \mathbb{R} \). In defocusing case we consider the cubic-quintic, \( V = \eta |\psi|^4 - \frac{\epsilon}{6} |\psi|^6, \eta > 0 \) and saturable \( \frac{dV(t)}{dt} = 2\eta |\psi|^2 - \frac{\epsilon|\psi|^6}{1+|\psi|^2}, \eta > 0, \epsilon \in \mathbb{Z}_+ \), potentials. However, for two-soliton field components without definite parity we also show numerically the vanishing of the first non-trivial anomaly and the exact conservation of the relevant charge. So, the parity symmetry seems to be a sufficient but not a necessary condition for the existence of the infinite tower of conserved charges in the both type of models. The model supports elastic scattering of solitons for a wide range of values of the amplitudes and velocities and the deformation parameter \( \epsilon \). Since the NLS equation is ubiquitous, our results may find potential applications in several areas of non-linear science.

References

2. H. Blas and M. Zambrano, JHEP03(2016)005